Problem 4: よ-Actions - Solution

We note that there is a bijection between intertwiners $f : G \to X$ and elements of X. Let us denote the set of intertwiners by $Hom_G(G, X)$. Then, we have the following maps

$$\begin{aligned} Hom_G(G, X) &\to X \\ f &\mapsto f(e) \end{aligned}$$

and

$$\begin{aligned} X &\to Hom_G(G, X) \\ x &\mapsto f_x : G \to X \end{aligned}$$

with $f_x(g) = gx$. Note $f_x(hg) = hgx = hf_x(g)$, so f_x is an intertwiner and $x \mapsto f_x \mapsto f_x(e) = x$ and $\alpha \mapsto \alpha(e) \mapsto f_{\alpha(e)}$. But $f_{\alpha(e)}(g) = g\alpha(e) = \alpha(g)$. So indeed a bijection. Thus, we just have to print the size of X. This is actually a corollary of the Yoneda of lemma where we view a group as a category with 1 point.